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Transition from propagating localized states to spatiotemporal chaos in phase dynamics

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We study the nonlinear phase equation for propagating patterns. We investigate the transition from a propagating localized pattern to a space-filling spatiotemporally disordered pattern and discuss in detail to what extent there are propagating localized states that breathe in time periodically, quasiperiodically, and chaotically. Differences and similarities to the phenomena occurring for the quintic complex Ginzburg-Landau equation are elucidated. We also discuss for which experimentally accessible systems one could observe the phenomena described. [S1063-651X(98)51910-1]

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Recently it has become clear [1,2] that localized states that breathe periodically, quasiperiodically, or even chaotically in time can stably exist for the complex quintic Ginzburg-Landau equation in one and two spatial dimensions. This equation is of prototype character for pattern formation in dissipative nonequilibrium systems. For this equation the spatially localized state is embedded in a background corresponding to zero amplitude, which is one of the two locally stable states of the spatially homogeneous equation. As long as the perturbations propagating down the wings of the breathing localized states are sufficiently damped, the state stays localized. Larger dispersive contributions then lead to a growth of these perturbations and to a filling in of the entire cell [1].

Triggered by these results the question arises whether the behavior found for the complex quintic Ginzburg-Landau equation is very special or to what extent phenomena such as periodically and chaotically breathing localized states can also be found for other prototype equations. To address this question we have investigated the transition from a propagating localized solution with fixed shape to irregular behavior in space and time for the whole cell in the framework of the nonlinear phase equation for propagating phase motion.

In this context we denote by phase dynamics [3] the approach that represents the analog of hydrodynamics [4,5] for large aspect ratio pattern-forming nonequilibrium systems [6,7]. While there have been numerous studies, both experimental [8–11] and theoretical [3,7,12], on linearized phase dynamics, nonlinear phase dynamics has not yet attracted as much attention. For stationary patterns we have shown [13] that the appropriate nonlinear phase equation can have stationary localized states for which the wavelength of the pattern varies as a function of space: the wave vectors in a localized area over part of the cell are different from the constant background wavelength. The properties of these stationary confined states in phase dynamics, which are intrinsically nonlinear objects [13], have been studied in some detail theoretically [13–16]. On the experimental side there have been thorough investigations on slot convection far above onset [17–20], both for straight and annular cells, which reveal, among other results, phenomena that have many qualitative similarities with the localized states studied theoretically. For the nonlinear phase equation associated with propagating patterns [6] we have already shown [21] that propagating localized patterns with fixed shape exist stably and that there can be a transition to a space-filling pattern that is disordered in time and space, provided there are sufficiently large nonpotential contributions.

To investigate to what extent breathing localized states can arise in nonlinear phase dynamics, we start with the nonlinear phase equation describing the long wavelength, low frequency modulations of a propagating pattern in one dimension [22],

$$\dot{\phi}-v\,\phi_x-C\,\phi_{xxx}=[D+E\,\phi_x+F\,\phi_x^2]\,\phi_{xx}+(H/2)\,\phi_x^2 - G\,\phi_{xxxx}\,, \tag{1}$$

which reads when rewritten for the local wave vector $q=\phi_x$

$$\dot{q}-v\,q_x-Cq_{xxx}=[D+Eq+Fq^2]q_{xx}+[E+2Fq]q_x^2+Hqq_x - Gq_{xxxx}\,. \tag{2}$$

The terms $\propto$ D, E, F, and G are already known from the nonlinear phase equation for a stationary phase [13–15]. The term $\propto D$ corresponds to ordinary linear phase diffusion, the term $\propto G$ is a higher order gradient term necessary to stabilize the system for D close to 0 or negative, the contribution $\propto E$ is the lowest order nonlinearity for a stationary phase, and the one $\propto F$ is necessary to stabilize the system. We also note that the existence of these nonlinear terms makes two local minima of the associated potential possible in the stationary case [13]. The three additional terms in the case of a propagating phase correspond to the convective term (proportional to the group velocity v), its higher order gradient term ($\propto C$), and the nonlinear term ($\propto H$) characteristic of propagating phase motions, which has been studied by Kuramoto [23,24]. We note that the term $\propto H$ in Eq. (2) is of the form of a nonlinear gradient term and is similar to the convective nonlinearity of the one-dimensional Navier-Stokes equation.

Since we are studying here the transition to spatiotemporal disorder, only the case $D<0$ is relevant in the present

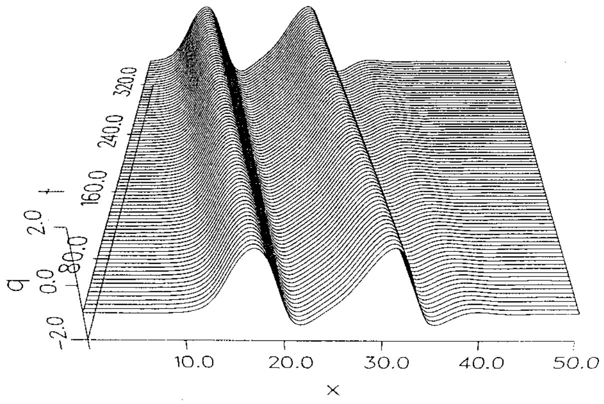

FIG. 1. Three-dimensional space-time plot for the wave vector q of a propagating localized state with fixed shape as it arises for $v=0$, $H=0.8$ ($D=-1, F=G=1, E=0$).

paper. By going into the frame moving with velocity v one can transform away the convective term in Eqs. (1 and 2).

In the following we describe the results of our numerical calculations, which have been obtained for periodic boundary conditions for the local wave vector q. We used a time-splitting method to integrate the linear part exactly in time using Fourier transforms and the nonlinear terms with second-order Runge-Kutta using fourth-order spatial differencing (compare, for example, Ref. [15] for more details of the numerical method). We were typically running our simulations for about 10^6 iterations in time (using a time step of $\Delta t=0.01$). Throughout the following we have transformed Eqs. (1) and (2) into the moving frame and we have put $C=0$ for simplicity.

In Fig. 1 we show for reference a space-time plot of a stable propagating localized state of fixed shape, which can be obtained, for example, from a Gaussian initial condition. We use a value of $H=0.8$, which represents a nonlinear gradient term in the equation for the local wave vector [Eq. (2)]. Inspection of Fig. 1 shows that this localized state is asymmetric (as is to be expected in the presence of a nonlinear gradient term) and travels to the left, while the shape is not changing as a function of time. We also note that the background wave vector is constant and shows no spatiotemporal variation.

To study the transition from the localized states of fixed shape to the space-filling pattern with spatiotemporal disorder, we have gradually increased the nonlinear gradient, nonpotential term, thus increasing the importance of the nonpotential term relative to the gradient parts, whose magnitude we keep fixed. In Fig. 2(a) we show a three-dimensional space-time plot for $H=1.4$. As we see from Fig. 2(a) a localized state results that is periodically breathing as a function of time. In addition we note that in contrast to the case of a propagating pulse of fixed shape, the background wave vector is no longer flat, but that small amplitude waves are being sent out that are reminiscent of the linear excitations (''phonons'' or ''radiation'') in solitonic systems. We stress that these excitations do not die out for the present system in the long-time limit, but rather that they are emitted periodically in time. We would like to emphasize that the onset of these excitations on the flat background as a function of H coincides with the onset of the breathing motion of the pulses. In Fig. 2(b) we have plotted the quantity S

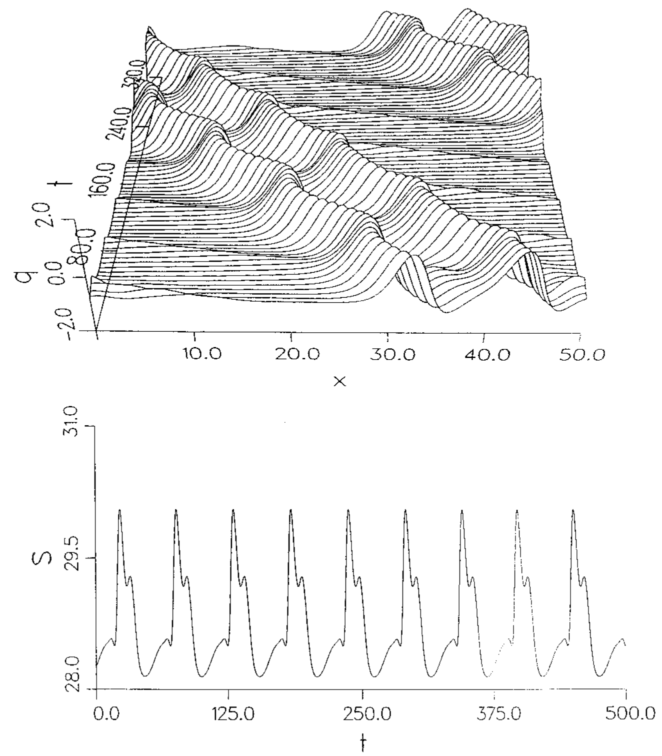

FIG. 2. (a) Three-dimensional space-time plot for a periodically breathing localized state is shown for $H=1.4$. We note the onset of the phonons (radiation) as soon as the propagating localized state starts to breath. (b) $\int q^2(x)dx$ shows periodic oscillations.

$=\int q^2 dx$. It is seen to show periodic oscillations. The quantity S can be thought of as a representative quantity showing oscillations and it has been selected for display, since for phase equations the integral over the wave vector q vanishes.

As the value of H is increased the speed at which the localized object is traveling also increases. Eventually we observe a transition from a periodic breathing to a quasi-periodic breathing of the localized state. This is shown in Fig. 3. In Fig. 3(a) we show a space-time plot for $H=2.2$. Inspecting this figure it is clear that the shedding of small amplitude excitations continues, although there is no longer any detectable periodicity associated with this process. In Fig. 3(b) and 3(c) we have plotted the logarithm of the separation $\zeta=\sqrt{\int \delta q^2 dx}$ (where δq is a linear perturbation about q) [25], and the integral quantity S, respectively. We see that the separation levels off in the long-time limit. This clearly demonstrates that this state is not chaotic. From Fig. 3(c) we infer that the state considered also shows no detectable periodicity. Combining all our data, we conclude that this state is quasi-periodic.

A further increase in the magnitude of the nonlinear gradient term H leads to a transition from a quasi-periodic breathing localized state to a state that breathes chaotically and shows an increased frequency of low amplitude emissions, which appear to occur at random time intervals. To demonstrate that the localized state shown in Fig. 4(a) is chaotic, we plot in Fig. 4(b) the logarithm of the separation ζ in the long-time limit. We see that two nearby states separate exponentially on the average, verifying that the state is indeed chaotic.

As the value of H is increased further a transition to a

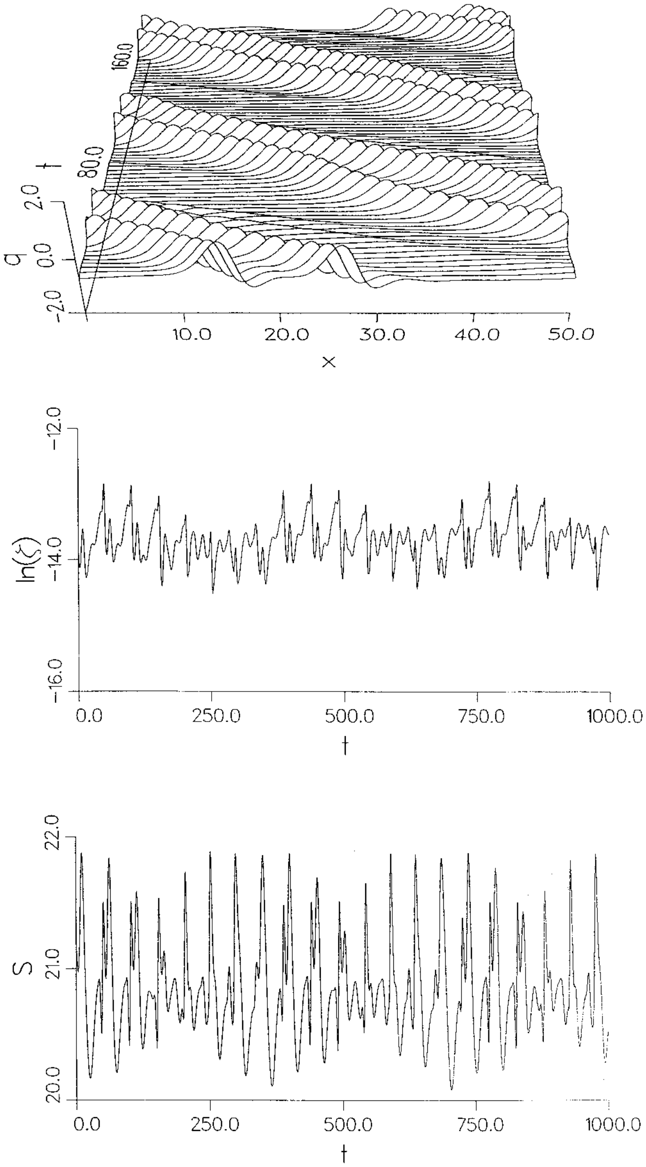

FIG. 3. (a) Space-time plot for a quasiperiodically breathing localized solution for $H=2.2$. We draw the attention to the higher speed with which the localized state propagates compared to the periodically breathing localized states. Note that the shedding of the linear excitations also does not show any obvious regularity anymore. (b) The separation shows that this state is not chaotic. (c) The integral S does not show any detectable periodicity.

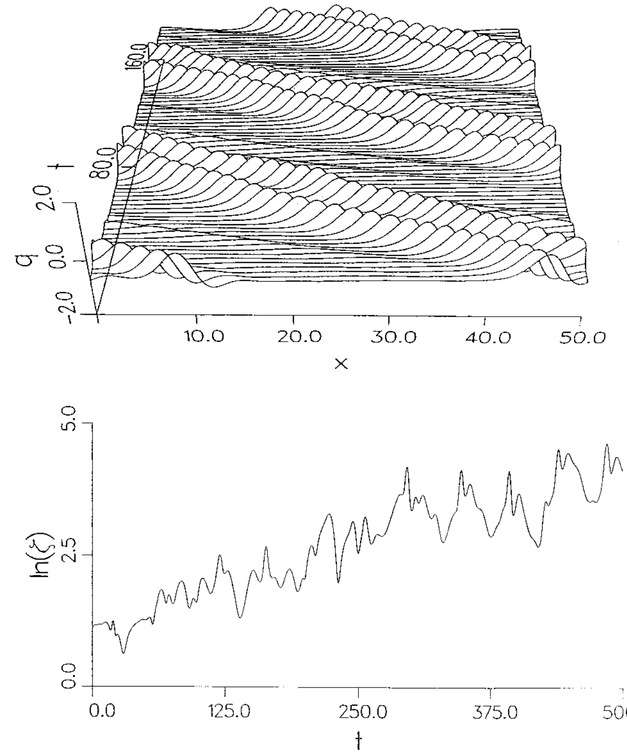

FIG. 4. Behavior of a chaotically breathing localized state for $H=2.8$. (a) The space-time plot shows an increased frequency of low amplitude emissions. (b) The separation clearly shows an exponential growth.

space-filling state that is disordered in time and space takes place [21]. Thus we see that we obtain the sequence localized state with fixed shape, periodically breathing localized state, quasi-periodically breathing localized state, chaotically breathing localized state, and eventually space-filling spatiotemporal disorder as H is increased. We emphasize that the breathing localized states for the nonlinear phase equation studied are always accompanied by the emission of small amplitude excitations (phonons, radiation). Their regularity and frequency also reflects the regularity and frequency of the underlying state. This is different from the case of the breathing localized states in the complex quintic Ginzburg-Landau equation, where there was no radiation sent off from the localized states in the long-time limit. It appears that this difference is related to the fact that the background has amplitude zero for the complex quintic Ginzburg-Landau equation, while there is a nonzero constant background value for the breathing localized states in nonlinear phase dynamics described here.

In this Rapid Communication we have shown that it is possible to get propagating breathing localized states in phase dynamics for traveling patterns: a localized excitation that travels and breathes has wavelengths that are different from that in the bulk of the pattern. As the control parameter is varied, the pulses in the wavelength distribution breathe periodically, quasi-periodically and eventually chaotically. Simultaneously with the onset of the breathing motion, we obtain the onset of the emission of small amplitude excitations (phonons, radiation), which are emitted periodically or chaotically even after a long time. Eventually (that is, at higher values of the nonlinear gradient term) a space-filling pattern that is disordered in space and time results.

Stimulated by the experimental results of Dubois and coworkers [17–20] on slot convection (in slot convection the width of the cell is much less that the height) in rectangular and annular slots in simple fluids, we speculate that one might see propagating confined states in this geometry for binary fluid mixtures. It is important to note in this connection that the experiments in Refs. [17–20] focused on simple fluids, while systematic experiments on binary fluid convection in the slot geometry have apparently not been performed as yet. In Refs. [17] and [20] a localized spatial variation of the pattern wavelength has been observed well above the instability onset, sometimes accompanied by temporal oscillations. Since we have focused on phase dynamics here, that is on the analog of hydrodynamics for large aspect ratio pattern forming systems, the scenario described is one of several possibilities. Alternative scenarios include the generation of spatiotemporal disorder via the generation of defects, which are characterized by space-time locations of vanishing amplitude. Other candidates of physical systems for which such phenomena might be observable include the spirals in the Taylor instability between counter-rotating cylinders [26], the flow between corotating cylinders (where domains in wavelength have been observed [27,28], and traveling waves in convective systems far above onset, such as binary fluid mixtures and electroconvection in nematic liquid crystals. As we have studied here spatial variations in only one spatial direction, clearly experimental setups involving quasi-one-dimensional geometries, such as annular shaped containers for thermal and electroconvection or Taylor cylinders approaching the narrow gap limit, will be more promising in observing the route to spatiotemporal chaos via breathing localized states described here.

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[1] R. J. Deissler and H. R. Brand, Phys. Rev. Lett. **72**, 478 (1994).
[2] R. J. Deissler and H. R. Brand, Phys. Rev. Lett. **74**, 4847 (1995).
[3] Y. Pomeau and P. Manneville, J. Phys. (France) Lett. **40**, 609 (1979).
[4] L. D. Landau and E. M. Lifshitz, *Hydrodynamics* (Pergamon, New York, 1959).
[5] D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions* (Benjamin, Reading, MA 1975).
[6] H. R. Brand, Prog. Theor. Phys. **71**, 1096 (1984).
[7] H. R. Brand, in *Propagation in Systems Far from Equilibrium*, edited by J. E. Wesfreid *et al.*, Springer Series in Synergetics Vol. 41 (Springer, Heidelberg, 1988), p. 206.
[8] J. E. Wesfreid and V. Croquette, Phys. Rev. Lett. **45**, 634 (1980).
[9] L. Ning, G. Ahlers, and D. S. Cannell, Phys. Rev. Lett. **64**, 1235 (1990).
[10] M. Wu and C. D. Andereck, Phys. Rev. Lett. **67**, 1258 (1991).
[11] M. Wu, C. D. Andereck, and H. R. Brand, Europhys. Lett. **19**, 587 (1992).
[12] H. Brand and M. C. Cross, Phys. Rev. A **27**, 1237 (1983).
[13] H. R. Brand and R. J. Deissler, Phys. Rev. Lett. **63**, 508 (1989).
[14] H. R. Brand and R. J. Deissler, Phys. Rev. A **41**, 5478 (1990).
[15] R. J. Deissler, Y. C. Lee, and H. R. Brand, Phys. Rev. A **42**, 2101 (1990).
[16] H. Riecke, Europhys. Lett. **11**, 213 (1990).
[17] M. Dubois, P. Bergé, and A. Petrov, in *New Trends in Nonlinear Dynamics and Pattern Forming Phenomena*, edited by P. Huerre and P. Coullet (Plenum, New York, 1991), p. 227.
[18] M. Dubois, R. DaSilva, F. Daviaud, P. Bergé, and A. Petrov, Europhys. Lett. **8**, 135 (1989).
[19] F. Daviaud, P. Bergé, and M. Dubois, Europhys. Lett. **9**, 441 (1989).
[20] J. Hegseth, J. M. Vince, M. Dubois, and P. Bergé, Europhys. Lett. **17**, 413 (1992).
[21] H. R. Brand and H. R. Deissler, Phys. Rev. A **46**, 888 (1992).
[22] H. R. Brand, Phys. Rev. A **31**, 3454 (1985).
[23] Y. Kuramoto and T. Tsuzuki, Prog. Theor. Phys. **54**, 687 (1976).
[24] Y. Kuramoto, Prog. Theor. Phys. Suppl. , **64**, 346 (1980).
[25] R. J. Deissler, J. Stat. Phys. **54**, 1459 (1989).
[26] C. D. Andereck, S. S. Liu, and H. L. Swinney, J. Fluid Mech. **164**, 155 (1986).
[27] G. W. Baxter and C. D. Andereck, Phys. Rev. Lett. **57**, 3046 (1986).
[28] C. D. Andereck and G. W. Baxter, in *Propagation in Systems Far From Equilibrium* (Ref. [7]), p. 315 ff.